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Holography of the $\mathcal{N}=1$ higher-spin theory on AdS$_4$

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Abstract: We argue that the $\mathcal{N}=1$ higher-spin theory on AdS$_4$ is holographically dual to the $\mathcal{N}=1$ supersymmetric critical $O(N)$ vector model in three dimensions. This appears to be a special form of the AdS/CFT correspondence in which both regular and irregular bulk modes have similar roles and their interplay leads simultaneously to both the free and the interacting phases of the boundary theory. We study various boundary conditions that correspond to boundary deformations connecting, for large-N, the free and interacting boundary theories. We point out the importance of parity in this holography and elucidate the Higgs mechanism responsible for the breaking of higher-spin symmetry for subleading N.

Keywords: Field Theories in Lower Dimensions, Conformal Field Models in String Theory, AdS-CFT and dS-CFT Correspondence, Supersymmetry and Duality.

Contents

1. Introduction and the Klebanov-Polyakov proposal

It has been long understood that consistent higher-spin gauge theories admit AdS spacetimes as vacua [1]. Nevertheless, only recently the question of the holography of higher-spin theories has been raised [2]. The interest in this holography currently is growing as it is gradually realized that it touches upon important issues such as the small tension limit of string theory and the holography of weakly coupled quantum field theories. (some other recent work includes [3].)

A concrete proposal for the holographic dual of a simple higher-spin theory was made in [4]. Consider the minimal bosonic higher-spin algebra in $d=4$

$$hs(4) \ni \mathrm{SO}(3,2)\,. \tag{1.1}$$

The unitary irreducible representations (UIR) of Spin$(3,2)$ are characterized by the quantum numbers of the subgroup SO$(2,1) \times$ SO$(1,1)$; they are labeled $D(\Delta, s)$, with Δ the dimension and s the total spin. The massless UIR's saturate the unitarity bound $\Delta \geq s+1$. (In addition, there are the exceptional UIR's $D(2,0)$, $D(1/2,0)$ and $D(1,1/2)$. The latter two are singletons.) The "currents" that can be obtained as symmetric composites of the basic scalar singleton UIR $D(1/2,0)$ have even spin $s = 0,2,4,\ldots$. This is the spectrum of the minimal bosonic higher-spin theory

$$[D(1/2,0) \otimes D(1/2,0)]_S = D(1,0) \oplus \sum_{s=1}^{\infty} D(2s+1, 2s)\,. \tag{1.2}$$

On AdS_4, the realization of this minimal higher-spin bosonic theory may be consistently constructed, although only partial information is explicitly available for the action of the theory. For each state in the spectrum, one considers a corresponding AdS_4 field. In particular, the $D(1,0)$ UIR is associated to a conformally coupled scalar on AdS_4

$$I_4 = \frac{1}{2\kappa_4^2} \int \mathrm{d}^4x \sqrt{-g} \left(-\frac{1}{2}(\partial h)^2 - \frac{1}{2} m^2 h^2 + \cdots \right) , \tag{1.3}$$

with $m^2 = -2$. Similar, yet largely unknown, terms should be written for the higher-spin UIR's.

The spectrum of the higher-spins is by construction in one-to-one correspondence with the spectrum of the conserved higher-spin currents in a free bosonic theory in $d = 3$. To study the holographic dual of (1.3) one can always normalize the 2-pt functions in the boundary to be of order 1, and then observe that the n-pt correlation functions for $n \geq 3$ are proportional to $(2\kappa_4^2)^{n/2}$, i.e., are suppressed by powers of the Planck length in AdS$_4$. This may be taken to imply that the elementary fields in the holographic dual of the minimal bosonic higher-spin theory carry a certain group representation. It was observed in [4] that the elementary fields should carry a vector representation, rather than adjoint, in order that the composite singlet currents reproduce the $hs(4)$ spectrum (1.1) and the concrete proposal is that the boundary theory is the $O(N)$ bosonic vector model in $d = 3$. This essentially means that one identifies the Planck length in AdS$_4$ with $1/\sqrt{N}$ as $2\kappa_4^2 \sim 1/N$, a normalization that we will adopt below.

Now, the mass of the conformally coupled scalar in (1.3) is such that both the *regular* and the *irregular* boundary modes could be used to construct a boundary effective action that gives positive definite 2-pt functions [5]. Then, one observes that the conformally coupled scalar in AdS$_4$ gives the "spin-zero" current of the free O(N) theory only if one uses the *non-standard* dimension Δ_-, which corresponds to the *irregular* mode. Using the *standard* dimension Δ_+ appears to give the large-N interacting fixed point of the model [4]. The existence of these two fixed points is related to the fact that the choice of either Δ_- or Δ_+ can be imposed by appropriate boundary conditions corresponding to "double-trace" deformations[1] of the boundary theory: the choice Δ_+ can be imposed

[1]With a slight abuse in terminology, "double-trace" operators here mean operators that are quadrilinear in the elementary $O(N)$ vector fields.

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by a relevant "double-trace" deformation of the boundary UV fixed point and the choice Δ_- can be imposed by an irrelevant deformation of the IR boundary fixed point. It is interesting to note that the existence of the two different fixed points in the boundary is also tied to the existence of a large-N limit. [6, 7]

2. A fermionic realization of the IR boundary theory and the role of parity

There appears to be another possible construction of the Δ_+ theory above. Suppose we start with the fermionic singleton $D(1,1/2)$. We have

$$[D(1,1/2)\otimes D(1,1/2)]_A = D(2,0)\oplus\sum_{s=1} D(2s+1,2s)\,. \tag{2.1}$$

This product contains the $D(2,0)$ UIR plus the same tower as in (1.2). In this case it seems that the natural boundary theory to associate with this spectrum is a free $O(N)$ Majorana fermionic theory. Indeed, at leading order in $1/N$ the dimension of the basic $O(N)$-singlet $\bar\psi^a\psi^a$ current is one, as is the IR dimension of the current in the interacting boson model. This observation appears to suggest that at leading order in $1/N$ the interacting fixed point of the bosonic $O(N)$ model involves free fermions. Moreover, the recent calculation in [8] of the 3-pt functions of the "spin-zero" current in the critical $O(N)$ vector model at both its UV and IR fixed points for large-N might be viewed as additional support for such a claim. It was there found, (following earlier work in [9]), that the 3-pt function of the "spin-zero" current in the IR fixed point (i.e., the operator with dimension 2) vanishes. This is consistent with the fact that the 3-pt function of the current $\bar\psi^a\psi^a$ is zero in the free fermionic theory. Furthermore, one may consider the "double-trace" deformation $(\bar\psi\psi)^2$ of the free fermionic boundary theory. From an RG point of view this is an irrelevant deformation and therefore it is consistent with the fact that the free fermionic theory corresponds to an IR fixed point [10]. We can then ask what is the UV fixed point at the other end of this irrelevant deformation? The answer is that such a UV fixed point involves, at leading-N, exactly the spectrum (1.2) and therefore seems to correspond to the free $O(N)$ vector model.

The above picture is appealing but overlooks the subtle role of parity.[2] First, let us note that the Legendre transform in AdS/CFT is simply implementing the "intertwining" operation — i.e., interchanging representations of dimensions Δ with those of dimension $d-\Delta$ [11]. This is related to a conformal inversion of the form $x^\mu \to x^\mu/x^2$ which explains the UV–IR relationship. However, there is an additional discrete parity transformation (i.e., reflection in one of the spatial coordinates), necessary to bring the inversion into SO(3, 2) [12]. Starting with a bulk scalar as in (1.3) the two boundary theories corresponding to the choices Δ_- and Δ_+ are related by a Legendre transform [5] and hence the two different boundary UIRs $D(1,0)$ and $D(2,0)$ are both scalars i.e. they have positive parity. The crucial point is now that a free-field representation of (1.2) with elementary scalars is only possible when $D(1,0)$ has positive parity, while a free-field fermionic representation

[2]We are indebted to P. Sundell for extensive discussions that led to the clarification of the role of parity.

of (2.1) is possible when $D(2,0)$ has negative parity. Therefore, the fixed points corresponding to the Legendre transforms of the free bosonic and free fermionic theories do not appear to correspond to free field theories, even for large-N. It is intriguing, however, that the 3-pt function of the parity-even UIR $D(2,0)$ at the IR point of the $O(N)$ vector model vanishes.

3. $\mathcal{N}=1$ higher-spin theory in AdS$_4$ and the $\mathcal{N}=1$ $O(N)$ vector model in $d=3$

Our aim here is to discuss the holography of the $\mathcal{N}=1$ supersymmetric higher-spin theory and show that the proposal of [4] and its intriguing properties arise as special cases. Supersymmetric versions of higher spin theories have been recently constructed [3]. The $\mathcal{N}=1$ theory $hs(1,4)$ is built from the two singleton UIR's $D(1/2,0)$ and $D(1,1/2)$ and its spectrum is given by

$$\begin{aligned}
[D(1/2,0)\otimes D(1/2,0)]_S &= D(1,0)\oplus\sum_{s=1} D(2s+1,2s)\,, &(3.1)\\
[D(1,1/2)\otimes D(1,1/2)]_A &= D(2,0)\oplus\sum_{s=1} D(2s+1,2s)\,, &(3.2)\\
D(1,1/2)\otimes D(1/2,0) &= D(3/2,1/2)\oplus\sum_{s=0} D(5/2+s,3/2+s)\,. &(3.3)
\end{aligned}$$

Given the successes of the bosonic theory, it is not hard to suggest that

> The minimal $\mathcal{N}=1$ higher-spin theory on AdS_4 is dual to the singlet part of the $\mathcal{N}=1$ supersymmetric $O(N)$ vector model in $d=3$.

The spectrum of the $\mathcal{N}=1$ minimal higher-spin theory is arranged into massless $Osp(1|4)$ supermultiplets [13]. There is one massless[3] "Wess-Zumino" multiplet that contains *both* scalar UIR's $D(1,0)$ and $D(2,0)$. In the AdS_4 realization of the theory these correspond to two conformally coupled scalars $h_1^{(+)}$ and $h_2^{(-)}$. The supermultiplet is completed by a bulk fermion field Ψ, the $D(3/2,1/2)$ UIR. One can easily write the quadratic part of the action for that multiplet as [14] (see also [15] for a related discussion)

$$I_4 = N\int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}\left(\partial h_1^{(+)}\right)^2+\left(h_1^{(+)}\right)^2-\frac{1}{2}\left(\partial h_2^{(-)}\right)^2+\left(h_2^{(-)}\right)^2+\frac{1}{2}\bar{\Psi}\not{D}\Psi+\cdots\right). \tag{3.4}$$

The dots on the right hand side of (3.4) corresponds to the kinetic terms of higher-spin fields as well as interactions. These are computable, at least in principle. The scalars $h_1^{(+)}$ and $h_2^{(-)}$ are real while the fermion Ψ is Majorana. Our notation is explained fully in the appendix.

[3] Massless refers to the fermion; the bosons are conformally coupled with $m^2=-2$.

Notice now that, as also explained in the appendix, the UIR's $D(1,0)$ and $D(2,0)$ in (3.4) have opposite parity. [16] This is not explicit in the bulk action (3.4), but is implied by supersymmetry (and consequently by boundary conditions at $r \to 0$ necessary to preserve supersymmetry). By parity (which could also be referred to as a discrete chiral symmetry), we mean a discrete element which we take to be $(x^0, x^1, x^2, x^3) \to (x^0, x^1, -x^2, x^3)$. Without loss of generality, we choose hereafter to assign positive parity to $h_1^{(+)}$ and negative parity to $h_2^{(-)}$ as indicated by the superscripts. Moreover, the two two-component Majorana spinors inside Ψ transform under parity with opposite signs.

We see then that the choice of boundary conditions determines essentially the nature of the boundary theories dual to (3.4). One of the two possible duals is the free $\mathcal{N}=1$ $O(N)$ vector model in $d=3$ whose action in superspace is given by (A.11). In this case, the bulk higher-spin currents in AdS_4 correspond to $O(N)$-singlet bilinears of the elementary boundary superfield Φ^a that may be represented as $\Phi^a D_{(i_1} \dots D_{i_{4n})} \Phi^a$, where the on-shell real superfield is given by

$$\Phi^a(x,\theta) = \varphi^a(x) - \bar{\theta}\psi^a(x)\,, \qquad a = 1,2,\dots,N\,. \tag{3.5}$$

In particular, the terms in the bulk action depicted in (3.4) should reproduce holographically the generating functional for correlation functions of the "spin-zero" current J, which is, on-shell

$$J(x,\theta) = \frac{1}{2}\Phi^2(x,\theta) = \frac{1}{2}\left(\varphi^a\varphi^a\right)(x) - \bar{\theta}\left(\psi^a\varphi^a\right)(x) - \frac{1}{2}\bar{\theta}\theta\left(\bar{\psi}^a\psi^a\right)(x)\,. \tag{3.6}$$

In the next section, we consider further the role of boundary conditions in the holography of the $\mathcal{N}=1$ $O(N)$ theories.

4. Boundary conditions and deformations: general remarks

The $\mathcal{N}=1$ supersymmetric $O(N)$ vector model in the boundary has been studied extensively in the literature using mainly a σ-model approach [17]. It possesses two critical points at large-N, one free and one interacting, which have various supersymmetric and non-supersymmetric deformations that could be considered. We consider several such deformations in section 5, although the "double-trace" deformations of the supersymmetric fixed points are of particular interest. From the bulk point of view, these correspond to suitable choices of the boundary conditions for scalars and spinors. We review this in the present context below [18, 19].

4.1 Scalars

We consider the conformally coupled scalars in (3.4) whose asymptotic behavior near the AdS_4 boundary at $r \to 0$ is

$$h_k^{(\pm)} = r\alpha_k^{(\pm)}(x) + r^2\beta_k^{(\pm)}(x) + \cdots\,, \qquad k = 1,2\,. \tag{4.1}$$

Requiring that the bulk solutions vanish as $r \to \infty$ one obtains

$$\alpha_1^{(+)}(x) = -\frac{1}{2\pi^2}\int d^3y\,\frac{1}{(x-y)^2}\beta_1^{(+)}(y)\,, \tag{4.2}$$

$$\beta_2^{(-)}(x) = -\frac{1}{2\pi^2}\int d^3y\,\frac{1}{(x-y)^4}\alpha_2^{(-)}(y)\,. \tag{4.3}$$

Therefore, each bulk solution (4.1) depends on one arbitrary function and when substituted back into the bulk action yields a well defined boundary functional. Usually one is forced to consider a functional of the α's and this is referred to as choosing the *regular* boundary conditions. However, as pointed out in [5] there are cases where a functional of the β's is perfectly acceptable, and this is referred to as choosing the *irregular* boundary conditions. Then, the functionals of the α's and the β's are related by a Legendre transform as is implied by (4.2) and (4.3), which preserves the parity assignments. Furthermore, choosing as boundary sources one of the α's or the β's, the other becomes the expectation value of the boundary operator. Explicitly, to get the free boundary fixed point we want here to choose $\beta_1^{(+)}(x)$ as a source for the boundary operator $J_1^{(+)}(x)$, and then $\alpha_1^{(-)}(x)$ represents the one-point function $\langle J_1^{(-)}(x)\rangle$. Conversely, for $J_2^{(-)}(x)$ it is $\alpha_2^{(-)}(x)$ that sources the field, and $\beta_2^{(-)}(x)$ gives the corresponding one-point function. Thus the supersymmetric boundary condition on the bulk scalar fields of the action (3.4) should be

$$\alpha_1^{(+)}(x) = \beta_2^{(-)}(x) = 0\,. \tag{4.4}$$

Now, if we want to perturb the lagrangian of the boundary theory, we take a suitable boundary condition for $h_k^{(\pm)}$. The precise form of the boundary condition will determine the actual perturbation. One simple deformation is the "single trace" $\int d^3x f_1^{(+)}(x)J_1^{(+)}(x)$, whereby we simply set the value of $\beta_1^{(+)} = f_1^{(+)}$. Similarly, if we want the single trace deformation $\int d^3x f_2^{(-)}(x)J_2^{(-)}(x)$, the appropriate boundary condition is $\alpha_2^{(-)} = f_2^{(-)}$. For more general deformations the prescription given by Witten [18] may be summarized as follows. If we wish to generate the perturbation

$$\int d^3x\, W\left(x; J_1^{(+)}, dJ_1^{(+)},\ldots, J_2^{(-)}, dJ_2^{(-)},\ldots\right), \tag{4.5}$$

we impose the boundary conditions

$$\beta_1^{(+)} = \frac{\delta W(x;\alpha_1^{(+)}, d\alpha_1^{(+)},\ldots,\beta_2^{(-)}, d\beta_2^{(-)},\ldots)}{\delta\alpha_1^{(+)}}\,, \tag{4.6}$$

$$\alpha_2^{(-)} = \frac{\delta W(x;\alpha_1^{(+)}, d\alpha_1^{(+)},\ldots,\beta_2^{(-)}, d\beta_2^{(-)},\ldots)}{\delta\beta_2^{(-)}}\,. \tag{4.7}$$

For completeness, let us give an example of how this result comes about. Consider for simplicity a single bosonic field h conjugate to the boundary operator $\mathcal{O}$ with dimension $1/2 < \Delta < 3/2$. The field h has the behavior

$$h \sim r^{3-\Delta}\alpha_0(x) + r^{\Delta}\beta_0(x) + \cdots\,, \tag{4.8}$$

while α_0 and β_0 are related as

$$\beta_0(x) \equiv \langle \mathcal{O} \rangle_{\alpha,0} \sim \int d^3y \, \frac{1}{(x-y)^{2(3-\Delta)}} \alpha_0(y) \,. \tag{4.9}$$

Add a boundary interaction $\frac{f}{2N} \int d^3y \; \mathcal{O}(y)^2$ and consider the calculation of

$$\langle \mathcal{O}(x) \rangle_{\alpha,f} = \left\langle \mathcal{O}(x) e^{i \frac{f}{2N} \int d^3y \; \mathcal{O}(y)^2} \right\rangle_{\alpha,0} . \tag{4.10}$$

We can proceed by expanding the exponential. The crucial point here is the assumption of a large-N expansion such that only the leading term in the OPE $\mathcal{O}(x)\mathcal{O}(y) \sim \frac{N}{(x-y)^{2\Delta}} I + \cdots$ contributes in the large-N limit. Then we derive

$$\begin{aligned} \langle \mathcal{O}(x) \rangle_{\alpha,f} &= \langle \mathcal{O}(x) \rangle_{\alpha,0} + f \int d^3y \, \frac{1}{(x-y)^{2\Delta}} \langle \mathcal{O}(y) \rangle_{\alpha,0} + \\ &+ f^2 \int d^3y \, \frac{1}{(x-y)^{2\Delta}} \int d^3z \, \frac{1}{(y-z)^{2\Delta}} \langle \mathcal{O}(z) \rangle_{\alpha,0} + \cdots . \end{aligned} \tag{4.11}$$

Now, we can multiply (4.11) by the inverse kernel of (4.9) to obtain

$$\int d^3x \, \frac{1}{(y-x)^{2(3-\Delta)}} \langle \mathcal{O}(x) \rangle_{\alpha,f} = \alpha_f(y) \,, \tag{4.12}$$

and we then have

$$\alpha_f(y) = \alpha_0(y) + f\beta_0(y) + f^2 \int d^3x \, \frac{1}{(y-x)^{2\Delta}} \beta_0(x) + \cdots \,, \tag{4.13}$$

and thus

$$\alpha_f = \alpha_0 + f\beta_f \,. \tag{4.14}$$

Setting then the sources to zero in the unperturbed theory $\alpha_0 = 0$, we arrive at the advertised boundary condition (4.6). It is clear that this derivation will generalize to an arbitrary functional W.

The derivation above elucidates also the meaning of choosing the *irregular* boundary conditions, i.e., considering a functional of β_0 in the case of (4.8). Formally, this corresponds to considering the Legendre transform of the standard functional of α_0 that is the generating functional of operators $\hat{\mathcal{O}}$ with dimension $3 - \Delta$. Consider now a boundary interaction $\frac{g}{2N} \int d^3y \; \hat{\mathcal{O}}(y)^2$. Following the same reasoning as above we find

$$\beta_g = \beta_0 + g\alpha_g \,. \tag{4.15}$$

We now see that $f \to \infty$ in the *regular* choice (4.14) leads to the unperturbed boundary condition $g = 0$ in (4.15) and hence to the *irregular* choice. The reverse also holds true and $g \to \infty$ in the *irregular* choice leads to the unperturbed $f = 0$ *regular* choice (4.14).

4.2 Spinors

Next we consider a massless bulk spinor Ψ. The Dirac equation has the form

$$\left[\Gamma^3\left(r\partial_r - \frac{3}{2}\right) + r\Gamma^\mu\partial_\mu\right]\Psi = 0\,. \tag{4.16}$$

This can be transformed into a second order equation whose general on-shell solution is

$$\Psi(x,r) = \int \frac{d^3p}{(2\pi)^3}\, e^{ip\cdot x} r^2 K_{1/2}(pr)\begin{pmatrix} a_+(p) \\ a_-(p)\end{pmatrix}\,, \tag{4.17}$$

where $a_\pm$ are two-component spinors. Substituting back into the first order equations, we find asymptotically[4]

$$a_- = i\underline{\not n}a_+\,, \qquad n_\mu = \frac{p_\mu}{p}\,. \tag{4.18}$$

The asymptotic behavior of (4.17) near the boundary is

$$\Psi(x,r) \sim r^{3/2}\begin{pmatrix} u_+(x) \\ u_-(x)\end{pmatrix}\,, \tag{4.19}$$

where

$$u_-(x) = +\int d^3y\, \underline{G}(x-y)\; u_+(y)\,, \tag{4.20}$$

$$\underline{G}(z) = -\underline{G}(z)^{-1} = i\int \frac{d^3p}{(2\pi)^3}\; e^{ip\cdot z}\frac{\underline{\not p}}{p} = -\frac{1}{\pi^2}\frac{\not x}{x^4}\,. \tag{4.21}$$

Notice now that under the parity transformation of (A.12), u_+ and u_- transform with different signs, while the kernel (4.21) remains invariant. The latter is actually proportional to the 2-pt function of the boundary fermions, and is sometimes referred to as an intertwiner [11].

Since the boundary action is of the form [20]

$$\int d^3x(\bar{u}_+u_+ - \bar{u}_-u_-)\,, \tag{4.22}$$

it appears that which of $u_\pm$ one calls $\langle\mathcal{O}_{3/2}\rangle$ and which one calls the source is a matter of choice. However, the supersymmetry structure indicates the proper interpretation. Recall that in the bulk, the supercharge splits as in (A.18)

$$Q = \begin{pmatrix} q \\ s\end{pmatrix}\,, \tag{4.23}$$

where q is the supercharge and s is the superconformal generator. After making a choice of parity assignments for the bulk scalars, e.g., assign negative parity to $h_2^{(-)}$, the supersymmetry in the bulk requires that $qh_2^{(-)} \sim \Psi$. (in the notation of the appendix, $h_2^{(-)} \sim j_2$). Now, we remember that to get the free $O(N)$ theory in the boundary, we use the *regular*

[4] The precise form is $\frac{(yD+1/2)K_{1/2}(y)}{K_{1/2}(y)}a_\mp = \mp iy\underline{\gamma}^\mu n_\mu a_\pm$.

boundary conditions for $h_2^{(-)}$. Therefore in this case we should identify u_+ with the vev and u_- with the source. The opposite will hold true when we want to find the strongly-coupled $O(N)$ boundary theory.

Consider then the classically marginal "double-trace" operator

$$e^{i\frac{E}{2N}\int d^3x\ \bar{\mathcal{O}}_{3/2}\mathcal{O}_{3/2}}\,. \tag{4.24}$$

To determine the boundary condition to which it corresponds we follow the arguments in the previous subsection and write

$$u_-^{(E)}(x) = u_-^{(0)}(x) + iE\int d^3y\ iG(x-y)u_-^{(0)}(y) + \cdots\,. \tag{4.25}$$

Assuming then an OPE of the form

$$\mathcal{O}^i_{3/2}(x)\mathcal{O}^j_{3/2}(y) \sim NiG^{ij}(x-y)I + \cdots\,, \tag{4.26}$$

and a large-N expansion we arrive at the condition

$$u_-^{(E)} = u_-^{(0)} + Eu_+^{(E)}\,. \tag{4.27}$$

Note now that (4.27) is a boundary condition that does not preserve parity, but $E\to\infty$ clearly corresponds to switching q and s, and thus we would expect a rearrangement of the supermultiplet, if the theory is supersymmetric at $E\to\infty$.

5. Boundary conditions and deformations: $\mathcal{N}=1$ higher-spin theory

Now we are ready to present our concrete proposal by considering various boundary conditions for the bulk fields that appear in the action (3.4). First we will consider the large-N duals of the bulk theory which means that we discuss only the tree level bulk action. In section 5.3, we will discuss the effects of bulk loops on the higher-spin gauge symmetry.

5.1 Large-N duals

5.1.1 The free $\mathcal{N}=1$ supersymmetric $O(N)$ vector model

To obtain the free $\mathcal{N}=1$ Supersymmetric $O(N)$ vector model in the boundary we consider the *irregular* boundary condition for $h_1^{(+)}$ and the *regular* one for $h_2^{(-)}$. Making also an appropriate choice for the boundary condition of the bulk fermion, as explained in section 4.2, we preserve supersymmetry. Moreover, as we will argue in section 5.3 these boundary conditions will not break the higher-spin gauge symmetry even after bulk loops are taken into account.

5.1.2 The strongly coupled $\mathcal{N}=1$ supersymmetric $O(N)$ vector model

In this case, we use the *regular* boundary condition for $h_1^{(+)}$ and the *irregular* one for $h_2^{(-)}$. Then, with the opposite choice from above for the boundary conditions of the bulk fermions we still preserve supersymmetry. However, now the boundary "spin-zero" current multiplet cannot be represented by free fields due to the specific parity assignments. This

$\mathcal{N} = 1$ theory corresponds to the large-N limit of the IR fixed point of the $O(N)$ vector model. The reason is that, as we will argue in section 5.3, the specific boundary conditions chosen here will break the higher-spin gauge symmetry for subleading-N when bulk loops are taken into account. In this way the currents in the boundary theory will acquire anomalous dimensions of order $1/N$.

5.2 Deformations

Next, we consider deformations of the boundary lagrangian.

5.2.1 Mass deformations

The simplest deformation of the free $\mathcal{N} = 1$ theory that one could consider is a boundary condition corresponding to adding a mass term for elementary fields in the boundary theory. For example, a boundary fermion mass term will clearly lead to an infrared theory containing only currents built out of the elementary bosons φ^a, once $1/N$ corrections are taken into account. From the bulk point of view, this physics should be reproduced classically, by a suitable "domain wall". In particular, one must be able to see that all higher spin bulk fields that were coupled to boundary operators involving fermions (i.e., $h_2^{(-)}$ and Ψ and their higher-spin partners) are made massive by the choice of boundary condition. In this way, one is left with the spectrum of the $hs(4)$ bosonic higher spin theory. Similarly, a boundary boson mass term should leave only the fermionic currents in the boundary theory, or in the bulk, only $h_2^{(-)}$ and its higher-spin partners.

5.2.2 A marginal double-trace perturbation

Given the established connection between the Legendre transformation and double trace operators, it is clearly of interest to study these in the present context. There are two distinct choices that we will identify.

First, a (classically) marginal deformation of the free boundary theory is

$$\int d^3x \, \left\{ g_3 J_1^{(+)} J_2^{(-)} + g_4 \bar{J}_{3/2}^{(\pm)} J_{3/2}^{(\pm)} \right\} . \tag{5.1}$$

This is supersymmetric along the line $g_3 = -2g_4$ in which case it corresponds to the deformation $\int d^3x \, d^2\theta \, J^2$. It is easily seen that this deformation violates parity [21]. It is interesting to ask where the deformation (5.1) leads the free $O(N)$ vector model to, for large values of the couplings g_3, g_4 and large-N. One way to answer this is to recall that (5.1) can actually be imposed via an appropriate boundary condition on the bulk fields. In the notation of the previous section this is

$$\alpha_2^{(-)} = g_3 \alpha_1^{(+)} , \qquad \beta_1^{(+)} = g_3 \beta_2^{(-)} , \qquad u_- = -g_4 u_+ . \tag{5.2}$$

We now see that in the supersymmetric case the limit of large coupling constant and large-N just leads to the strongly coupled theory of subsection 5.1.2. Thus, even though the deformation (5.1) breaks parity, at large-N we end up with an $\mathcal{N} = 1$ supersymmetric theory.

It is also possible that the deformation (5.1) is actually *exactly* marginal. This sort of possibility was mentioned in ref. [18] in the bosonic theory, where it was noted that AdS_4 apparently remains a solution for any value of the coupling $\int d^3x \mathcal{O}_{\Delta_-}\mathcal{O}_{\Delta_+}$. In the present case, the situation is even better: eq. (5.1) is a deformation of a *free* CFT and thus marginality can be investigated perturbatively. We are not aware of literature related directly to this question, but it should not be too difficult an issue to settle.

5.2.3 A supersymmetry breaking deformation

Another interesting "double-trace" deformation of the free theory is

$$\frac{1}{2}\int d^3x \left\{g_1(J_1^{(+)})^2 + g_2(J_2^{(-)})^2 + g_3\bar{J}_{3/2}^{(\pm)}J_{3/2}^{(\pm)}\right\}. \tag{5.3}$$

This is a closer analogue to the double trace deformation considered for the bosonic theory. It corresponds to the boundary conditions

$$\alpha_2^{(-)} = g_2\beta_2^{(-)}\,, \qquad \beta_1^{(+)} = g_1\alpha_1^{(+)}\,, \qquad u_- = -g_3 u_+\,, \tag{5.4}$$

which of course break supersymmetry. g_1 corresponds to a relevant deformation of the free $O(N)$ fixed point while the g_2 deformation looks irrelevant from an RG point of view. Again, we can ask where the deformation (5.3) leads the free boundary theory to, for large values of the couplings and large-N. This can be answered using information from the bulk. Namely, we see from the boundary conditions (5.4) that for large values of the coupling constants one is apparently led to the strongly coupled theory of subsection 5.1.2 again! This time, the boundary conditions for the scalars are parity preserving while the one for the fermion is parity non-preserving. The remarkable result is that despite the fact the the deformation (5.3) breaks supersymmetry, at large-N we recover an $\mathcal{N}=1$ supersymmetric theory, at its strongly coupled fixed point. This is perhaps not as surprising as it seems, as the RG interpretation of the deformation (5.3) is rather unusual. Indeed, due to the structure of the Wess-Zumino multiplet, one may view the free boundary theory as being "half at the UV fixed point" (the part involving the bosons) while the other half is at its "IR fixed point" (the part involving the fermions).

5.3 Subleading-N and the breaking of higher-spin gauge symmetry

In the previous subsection we argued that the tree level $\mathcal{N}=1$ higher-spin theory on AdS_4 leads to *two* boundary 3d CFTs whose composite operators have *exactly* the same spectrum of dimensions. The only property that distinguishes the two boundary theories, at large-N, is the parity assignment of the operators in the supermultiplets. One choice of parity assignments leads to the free $O(N)$ vector model while the other choice leads to a strongly coupled version of the $O(N)$ vector model. The generating functionals of the two theories are related by a Legendre transform.

The distinction between the two theories should become more evident when considering bulk loop corrections or, equivalenty corrections subleading in $1/N$ in the boundary theory. In other words, we expect that starting with the boundary action (3.4) and considering the boundary conditions that lead to the free boundary theory, bulk loops do not break the higher-spin gauge symmetry. That is, the bulk higher-spin fields remain massless while the corresponding boundary currents remain conserved. On the other hand, considering the boundary conditions that lead to the strongly coupled boundary theory we expect that the bulk loops will render the higher-spin fields massive (Higgsing) and the corresponding boundary currents non-conserved [22].

The mechanism by which the phenomenon described above takes place is a generalization of the mechanism discussed in [23] for the case of the minimal bosonic higher-spin theory on AdS_4. The basic physics arises from the fact that a representation that is massive from the bulk point of view satisfies

$$D(\Delta, s) \to D(s+1, s) \oplus D(s+2, s-1) \tag{5.5}$$

as $\Delta \to s+1$. This can be taken to imply that in order for a spin s field to become massive, there must be a suitable Goldstone mode transforming as $D(s+2, s-1)$, it must be of the correct parity, and there must be a suitable coupling present in the bulk effective action.

Here we elucidate this argument further by discussing it from the boundary point of view. Conformal invariance requires that a boundary spin s current $J^{\mu_1,\dots,\mu_s}$ with dimension $\Delta_s = s+1$ is also conserved. Non-conservation of this current appears in the form of an anomalous dimension, $\Delta_s \to s+1+\gamma$. Let us assume that $\gamma \sim O(1/N)$. Then, the non-conservation of a boundary current means that there is an operator equation of the form

$$\partial_{\mu_1} J^{\mu_1,\dots,\mu_s}(x) \sim \frac{1}{\sqrt{N}} T^{\mu_2,\dots,\mu_s}(x)\,. \tag{5.6}$$

The current $T^{\mu_2,\dots,\mu_s}(x)$ has dimension $s+2$ to leading order in $1/N$. For non-coincident points, (5.6) leads to

$$\langle \partial_{\mu_1} J^{\mu_1,\dots,\mu_s}(x_1)\, \partial_{\nu_1} J^{\nu_1,\dots,\nu_s}(x_2)\rangle \sim \frac{1}{N} \langle T^{\mu_2,\dots,\mu_s}(x_1) T^{\nu_2,\dots,\nu_s}(x_2)\rangle\,. \tag{5.7}$$

Then, conformal invariance determines [24] the form of both sides in (5.7) and a tree-level calculation of the rhs of (5.7) yields the $1/N$ result for γ. What is crucial for us is that equations such as (5.6) can exist in the theory only if a current $T^{\mu_2,\dots,\mu_s}(x)$ with the appropriate dimension $s+2$ and parity can be constructed.

Now let us consider the $\mathcal{N}=1$ higher-spin theory on AdS_4. In the bulk effective action, there might be terms of the schematic form[5]

$$\frac{1}{\sqrt{N}} W^{a_1,\dots,a_s} W_{a_3,\dots,a_s} \partial_{a_1} \partial_{a_2} h\,, \tag{5.8}$$

where W are higher-spin currents and h is either of the two conformally coupled bulk scalars. If such a term exists, then it can give rise to a boundary 3-pt function of the form

$$\langle \partial_{\mu_1} J^{\mu_1,\dots,\mu_s}(x_1) J_{\mu_3,\dots,\mu_s}(x_2) \partial_{\mu_2} J(x_3)\rangle\,. \tag{5.9}$$

[5]These couplings should be suitably supersymmetrized, but we will not consider the details of this here.

This 3-pt function will be non-zero and would correspond to (5.7) if in the OPE of $J_{\mu_3,\ldots,\mu_s}$ with J there exists an operator such that its derivative produces the current $T^{\mu_2,\ldots,\mu_s}(x)$. Let us study this in more detail. The OPE in question is of the form

$$J_{\mu_3,\ldots,\mu_s}(x_2)J(x_3) \approx \mathcal{T}_{\mu_3,\ldots,\mu_s}(x_3) + (x_{23})^\nu \mathcal{S}_{\nu\mu_3,..\mu_s}(x_3) + \cdots , \tag{5.10}$$

where the dots stand for higher descendants and other operators. Now we have to take into account the dimension of $J(x)$. When $J(x)$ has dimension 2 the operator $\mathcal{T}$ in (5.10) has dimension $s+1$ and its derivative has dimension $s+2$ and could be a candidate for T. But only when J has matching parity can this OPE produce the correct T. On the other hand, when J has dimension 1, it is the operator $\mathcal{S}$ in (5.10) that has dimension $s+1$ and therefore its derivative might give rise to T. In this case however, only when J has opposite parity can the correct T arise. Moreover, it is easy to see that when $s = 2$ the OPE (5.10) is between the same boundary scalar. Therefore, whichever boundary condition one chooses, the correct T can never arise. Therefore, the boundary energy momentum tensor remains always conserved as it should.

6. Summary and discussion

We have presented a concrete proposal for the holographic dual of the $\mathcal{N} = 1$ higher-spin gauge theory on AdS$_4$. We have argued that the boundary theory is the $\mathcal{N} = 1$ supersymmetric $O(N)$ vector model in three dimensions. Both regular and irregular bulk modes are necessary for this holography and their interplay unveils interesting phenomena. In particular, the unique bulk theory gives rise to two boundary theories that are the free and interacting fixed points of the $O(N)$ vector model. At large-N, the boundary theories are distinguished only by the parity assignments in the supermultiplets. For subleading-N, only the boundary conditions that give the interacting $O(N)$ vector model in the boundary will Higgs the massless higher-spins and give rise to a boundary theory where all higher-spin currents acquire anomalous dimensions.

We studied various boundary conditions that correspond to "double-trace" deformations of the free boundary theory. Particularly intriguing is the fact that supersymmetry breaking boundary deformations lead for large-N to a supersymmetric theory. This phenomenon is tied to the fact that the free boundary theory may be viewed as being "half in a UV fixed point and half in an IR fixed point." We expect that a similar phenomenon occurs in the case of the the $\mathcal{N} = 1$ SCFT in four dimensions obtained holographically from the compactification of IIB SUGRA on AdS$_5\times$T^{1,1} [25]. We have also noted the possible existence of a line of fixed points in this model.

One would like to think that some of the salient features of this special holography are connected with the higher-spin gauge symmetry. In particular the fact that this holography gives rise to a free boundary theory is presumably a feature of higher-spin theories. Nevertheless, it appears that once we have information about the free boundary theory we also have information about an interacting boundary theory. This follows from the fact that the higher-spin multiplet includes simultaneously "shadow" UIRs of the conformal group and therefore describes at the same time UV and IR properties of the boundary

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theory. It would be interesting to study further our proposal and discuss supermultiplets containing currents with higher spins, in particular the energy momentum tensor. It is also be of interest to study the thermodynamics and the $O(N)$ symmetry breaking pattern of the boundary theory from the bulk point of view.

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A. Conventions

There is a real basis of generators for the $(-++)$ Clifford algebra

$$\underline{\gamma}^\mu = \{i\sigma_2, \sigma_1, \sigma_3\}\,. \tag{A.1}$$

Because of the reality of the generators of the Clifford algebra, we can take Majorana spinors satisfying

$$\psi = \hat{c}\bar{\psi}^T\,. \tag{A.2}$$

We can take $\hat{c} = \underline{\gamma}^0 \equiv \epsilon^{\alpha\beta}$, and thus the Majorana condition just reduces to $\psi^* = \psi$. Because of the choice of basis for the Clifford algebra, we have equations like

$$\bar{\psi}\eta = \psi^T\gamma^0\eta = \psi_\alpha\epsilon^{\alpha\beta}\eta_\beta = -\epsilon^{\beta\alpha}\psi_\alpha\eta_\beta = -\psi\eta\,. \tag{A.3}$$

Using real Poincaré generators, the $\mathcal{N}=1$ supersymmetry generators can be taken to satisfy

$$\{q_\alpha, q_\beta\} = 2\left(\underline{\gamma}^\mu\underline{\hat{c}}^{-1}\right)_{\alpha\beta}P_\mu\,. \tag{A.4}$$

A superspace representation for these generators is

$$q_\alpha = -(\hat{c}^{-1})_{\alpha\beta}\frac{\delta}{\delta\theta_\beta} + \left(\underline{\gamma}^\mu\theta\right)_\alpha\partial_\mu\,. \tag{A.5}$$

The general $\mathcal{N}=1$ superfield is written

$$\Phi(x,\theta) = \varphi(x) - \bar{\theta}\psi(x) + \frac{1}{2}\bar{\theta}\theta F(x) \tag{A.6}$$

and the supersymmetry transformations are

$$\delta\varphi = \bar{\epsilon}\psi\,, \qquad \delta\psi = F\epsilon + \partial_\mu\varphi\underline{\gamma}^\mu\epsilon\,, \qquad \delta F = -\bar{\epsilon}\underline{\gamma}^\mu\partial_\mu\psi\,. \tag{A.7}$$

The "spin-zero" current is defined as

$$J = \frac{1}{2}\Phi^2 = \frac{1}{2}(\varphi\varphi) - \bar{\theta}\left[(\varphi\psi)\right] + \frac{1}{2}\bar{\theta}\theta\left[(\varphi F) - \frac{1}{2}(\bar{\psi}\psi)\right], \tag{A.8}$$

therefore on-shell we have

$$J_1^{(+)} = \frac{1}{2}(\varphi\varphi)\,, \qquad J_{3/2}^{(\pm)} = (\varphi\psi)\,, \qquad J_2^{(-)} = -\frac{1}{2}(\bar{\psi}\psi)\,. \tag{A.9}$$

Then, taking

$$D_\alpha = (\hat{c}^{-1})_{\alpha\beta}\frac{\delta}{\delta\theta_\beta} + (\underline{\gamma}^\mu\theta)_\alpha\partial_\mu\,, \tag{A.10}$$

we find the free $\mathcal{N}=1$ lagrangian as

$$\int d^2\theta\frac{1}{2}\overline{D\Phi}D\Phi = -\frac{1}{2}\left[\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \bar{\psi}\underline{\gamma}^\mu\partial_\mu\psi + F^2\right]. \tag{A.11}$$

Parity in three dimensions is defined as

$$\hat{P}\psi\left(x^0,x^1,x^2\right) = \eta\left(\Pi\cdot\psi\right)\left(x^0,x^1,-x^2\right), \quad \eta^2=1\,, \tag{A.12}$$

and one can verify that a suitable choice is $\Pi\equiv\gamma^2$. Then one easily finds that the scalar $J_2^{(-)}(x)$ in (A.9) is odd under parity while $J_1^{(+)}$ is even, as the superscripts indicate. In general, $\mathcal{N}=1$ supersymmetry requires that the two scalars in the "spin-zero" multiplet have opposite parity. However, only the specific assignments above lead to a free-field theory representation as in (A.8).

Now we would like to extend this to the AdS_4 bulk [14]. First, we take the AdS_4 metric in Poincaré coordinates

$$ds^2 = \frac{1}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right), \qquad \mu,\nu=0,1,2\,. \tag{A.13}$$

It is simpler to work in $\mathbb{R}^5$ with metric $g^{AB}=diag(--+++)$, $A,B=-1,0,1,2,3$ where $\mathrm{Spin}(3,2)$ acts linearly. The most convenient basis is

$$\Gamma^{-1} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \qquad \Gamma^\mu = \begin{pmatrix} \underline{\gamma}^\mu & 0 \\ 0 & -\underline{\gamma}^\mu \end{pmatrix} \qquad \Gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{A.14}$$

The $\mathrm{Spin}(3,2)$ generators in the spinor representation are $S^{AB}=\frac{1}{4}\left[\Gamma^A,\Gamma^B\right]$. Denoting by M^{AB} the $\mathrm{SO}(3,2)$ generators, we can introduce a supercharge Q_α (here $\alpha=1,\ldots,4$)

$$\left[M^{AB},Q_\alpha\right] = -(S^{AB})_\alpha{}^\beta Q_\beta\,. \tag{A.15}$$

In the given basis, $\hat{C}=\Gamma^{-1}\Gamma^0$. We have

$$\{Q_\alpha,Q_\beta\} = -2\left(S^{AB}\hat{C}^{-1}\right)_{\alpha\beta}M_{AB}\,. \tag{A.16}$$

The utility of the chosen basis is that the generator of $\mathrm{Spin}(2,1)\subset\mathrm{Spin}(3,2)$ splits

$$S^{\mu\nu} = \begin{pmatrix} \frac{1}{2}\underline{\gamma}^{\mu\nu} & 0 \\ 0 & \frac{1}{2}\underline{\gamma}^{\mu\nu} \end{pmatrix}, \tag{A.17}$$

and thus it is sensible to define

$$Q = \begin{pmatrix} q \\ s \end{pmatrix}. \tag{A.18}$$

We can then work out the superalgebra relations[6] (now $\alpha = 1, 2$)

$$[L_{\mu\nu}, q_\alpha] = -\frac{1}{2}\left(\underline{\gamma}_{\mu\nu} q\right)_\alpha \qquad [L_{\mu\nu}, s_\alpha] = -\frac{1}{2}\left(\underline{\gamma}_{\mu\nu} s\right)_\alpha \,, \tag{A.19}$$

$$[P_\mu, q_\alpha] = 0 \qquad [P_\mu, s_\alpha] = -\left(\underline{\gamma}_\mu s\right)_\alpha \,, \tag{A.20}$$

$$[K_\mu, q_\alpha] = +\left(\underline{\gamma}_\mu q\right)_\alpha \qquad [K_\mu, s_\alpha] = 0 \,, \tag{A.21}$$

$$[D, q_\alpha] = -\frac{1}{2} q_\alpha \qquad [D, s_\alpha] = +\frac{1}{2} s_\alpha \,, \tag{A.22}$$

and

$$\{q_\alpha, q_\beta\} = 2(\underline{\gamma}^\mu \underline{\hat{c}}^{-1})_{\alpha\beta} P_\mu \,, \tag{A.23}$$

$$\{s_\alpha, s_\beta\} = -2(\underline{\gamma}^\mu \underline{\hat{c}}^{-1})_{\alpha\beta} K_\mu \,, \tag{A.24}$$

$$\{q_\alpha, s_\beta\} = 2(\underline{\hat{c}}^{-1})_{\alpha\beta} D - (\underline{\gamma}^{\mu\nu} \underline{\hat{c}}^{-1})_{\alpha\beta} L_{\mu\nu} \,, \tag{A.25}$$

where we have written $\hat{C} = \left(\begin{smallmatrix} 0 & \underline{\hat{c}} \\ \underline{\hat{c}} & 0 \end{smallmatrix}\right)$. It is clear from the form of the algebra that Q can be taken to be Majorana. This condition is

$$Q = C\overline{Q}^T \,, \tag{A.26}$$

where $C = \hat{C}\Gamma^{-1}$, and the condition amounts to $q^* = q, s^* = s$.

The bulk "Wess-Zumino multiplet" has real scalar components j_0, j_2 and a Majorana spinor $j_{1\alpha}$. In terms of UIRs of $Osp(1|4)$ this is of the general form [13]

$$D(\Delta, 0) \oplus D(\Delta + 1/2, 1/2) \oplus D(\Delta + 1, 0) \,. \tag{A.27}$$

Supersymmetry acts on them as

$$q_\alpha j_2 = j_{1\alpha} \,, \tag{A.28}$$

$$q_\alpha j_{1\beta} = \left(\underline{\gamma}^\mu \underline{\hat{c}}^{-1}\right)_{\alpha\beta} \partial_\mu j_2 + \left(\underline{\hat{c}}^{-1}\right)_{\alpha\beta} j_0 \,, \tag{A.29}$$

$$q_\alpha j_0 = -\left(\underline{\gamma}^\mu \partial_\mu j_1\right)_\alpha \,. \tag{A.30}$$

On-shell, the last equation (A.30) is zero. This essentially means that q_α acts as a dimension lowering operator such that $\Delta_1 = \Delta_2 - 1/2$ and $\Delta_0 = \Delta_1 - 1/2$, where $Dj_x = \Delta_x j_x$. Relevant to us is the case $\Delta_0 = 1$ when j_0, j_2 and j_1 are respectively the two conformally coupled scalars $h_1^{(+)}$ and $h_2^{(-)}$ and the bulk spinor Ψ in (3.4) Again, $\mathcal{N} = 1$ SUSY requires that one assigns different parities to the two scalar components of the multiplet. This can be easily seen if one realizes from (A.23) and (A.29) that the operator that lowers the dimension of j_2 by unity is proportional to $\epsilon^{\alpha\beta} q_\alpha q_\beta$ which is odd under parity [26]. On the spinor, parity acts as in (A.12) where now $\Pi = \Gamma^2$. A mass term on AdS_4 is parity odd.

[6] As usual, define $D = M_{-1,3}, K_\mu = M_{3,\mu} + M_{-1,\mu}, P_\mu = M_{3,\mu} - M_{-1,\mu}, L_{\mu\nu} = M_{\mu\nu}$.

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